

## Solution

### Class 12 - Physics

#### Physics Class XII - 2020-21

#### Section A

1. The actual transfer of electrons from one body to the other is the basic cause of charging.

OR

We know that dielectric constant of a medium is

$$k = \epsilon_r = \frac{\epsilon}{\epsilon_0}$$

$$\therefore \epsilon = k\epsilon_0 = 1 \times 8.854 \times 10^{-12}$$
$$= 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$$

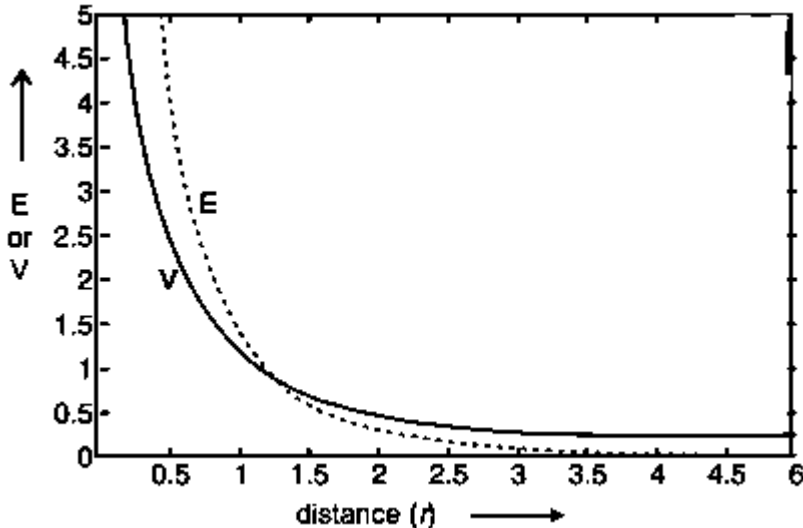
2. The occurrence of thunderstorms and lightning charges the atmosphere continuously. Hence, even with the presence of discharging current of 1800 A, the atmosphere is not discharged completely. The two opposing currents are in equilibrium and the atmosphere remains electrically neutral.
3. Electric field  $E$  due to a point charge  $Q$ ,

$$E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}, E \propto \frac{1}{r^2}$$

Electrostatic potential  $V$  due to a point charge  $Q$ ,

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \Rightarrow V \propto \frac{1}{r}$$

A graph showing variation of electric field ( $E$ ) and electric potential ( $V$ ) with distance ( $r$ ) are shown below:



4.  $W = q \times \Delta V$ . But  $\Delta V = 0$  for an equipotential surface,  
 $\therefore W = 0$
5. When the internal resistance of battery is zero, the potential difference is equal to emf.

OR

Yes, very slightly. The negatively charged body gains mass also along with electrons.

6. Since, electric field intensity inside the conductor is zero. So, electrostatic potential is constant.

$$\text{Mathematically, } E = -\frac{dV}{dr} = 0$$

$$\Rightarrow dV = 0 \dots(i)$$

Integrating the equation (i) we get  $V = \text{constant}$ .

Thus, the potentials at every point inside the conductor as well as on the surface of the conductor are same.

OR

The resistivity of a metallic conductor is given by

$$\rho = \rho_0 [1 + \alpha(T - T_0)]$$

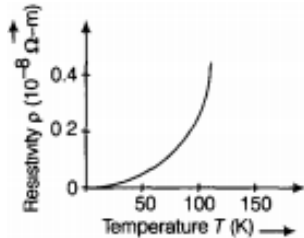
where,  $\rho_0$  = resistivity at reference temperature

$T_0$  = reference temperature and

$\alpha$  = coefficient of resistivity having unit /Kelvin.

From the above relation, we can say that the graph between resistivity of a conductor with temperature is straight line. But, at temperatures much lower than 273 K ( i.e.  $0^\circ\text{C}$ ), the graph deviates considerably from a

straight line as shown in the figure.



7. The fractional change in resistivity per unit change in temperature is called temperature coefficient and resistivity of a conductor increases with increase in temperature.

8. Siemen and Siemen metre<sup>-1</sup>

9. The reciprocal of resistivity of the material of a conductor is called its conductivity i.e.

$$\sigma = \frac{1}{\rho}$$

S.I. unit of  $\sigma$  is  $\Omega^{-1}m^{-1}$  or Siemen per metre.

10. The electron traces a circular path of radius,

$$r = \frac{1}{B} \sqrt{\frac{2mV}{q}}$$

where B is magnetic field, m is mass of proton, q is charge and V is potential difference.

When potential difference is doubled, the radius becomes,

$$r' = \frac{1}{B} \sqrt{\frac{2m(2V)}{q}} = \sqrt{2} r$$

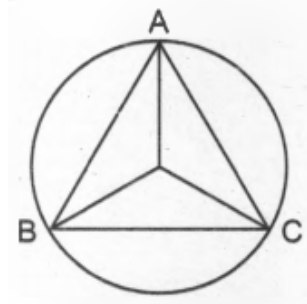
11. (c) A is true but R is false

**Explanation:** Energy conservation does not fail during the sharing of charges between two bodies. Some energy is lost in the form of heat or light or sparking.

12. (a) Both A and R are true and R is the correct explanation of A

**Explanation:**

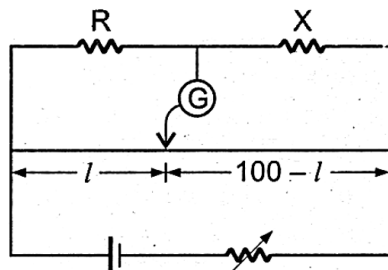
Resultant of electric intensity at O due to B and C is equal and opposite to that due to A



13. (d) A is false and R is also false

**Explanation:**

Both assertion and reason are false.



14. (a) Both A and R are true and R is the correct explanation of A

**Explanation:** Both A and R are true and R is the correct explanation of A

### Section B

15. i. (c) a vector quantity

ii. (b) cylindrical symmetric

iii. (a) C-m

iv. (a)  $10^{-10}$  C-m

v. (b) torque but no net force

16. i. (a) right  
 ii. (b) d.c bridge  
 iii. (c) quality control of wire  
 iv. (d) measure quantities such as capacitance, inductance and impedance  
 v. (a)  $\frac{R_1}{R_3} = \frac{R_2}{R_4}$

### Section C

17. Given, electric field intensity

$$\mathbf{E} = 5 \times 10^3 \mathbf{i} \text{NC}^{-1}$$

Magnitude of electric field intensity

$$|\mathbf{E}| = 5 \times 10^3 \text{NC}^{-1}$$

Side of square,  $S = 10 \text{ cm} = 0.1 \text{ m}$

Area of square,  $A = (0.1)^2 = 0.01 \text{ m}^2$

The plane of the square is parallel to the YZ-plane.

Hence, the angle between the unit vector normal to the plane and electric field is zero.

i.e.,  $\theta = 0^\circ$

$\therefore$  Flux through the plane is the scalar product of the Electric field vector and Area vector.

$$\phi = |\mathbf{E}| \times A \cos \theta \Rightarrow \phi = 5 \times 10^3 \times 0.01 \cos 0^\circ$$

$$\phi = 50 \text{Nm}^2 \text{C}^{-1}$$

If the plane makes an angle of  $30^\circ$  with the X-axis, then the angle between area vector and electric field will be,  $\theta = 60^\circ$

$\therefore$  Flux through the plane,

$$\phi = |\mathbf{E}| \times A \times \cos 60^\circ$$

$$= 5 \times 10^3 \times 0.01 \times \cos 60^\circ = 25 \text{Nm}^2 \text{C}^{-1}$$

Here the positive value of electric flux denotes that electric field lines are coming out of the loop.

OR

Given,  $\phi = -1.0 \times 10^3 \text{Nm}^2/\text{C}$

$r_1 = 0.1 \text{m}$ ,  $r_2 = 0.2 \text{m}$

- a. Doubling the radius of Gaussian surface will not affect the electric flux since the charge enclosed is the same in the two cases.

Thus, the flux will remain be the same i.e.  $-1.0 \times 10^3 \text{Nm}^2/\text{C}$

b.  $\phi = \frac{q}{\epsilon_0}$

$$\therefore q = \phi \cdot \epsilon_0$$

$$\text{or } q = -1.0 \times 10^3 \times 8.854 \times 10^{-12}$$

$$= -8.854 \times 10^{-9} \text{C}$$

$$= -8.854 \text{ nC}$$

Therefore, the value of the point charge is  $-8.854 \text{ nC}$ .

18. Work done in moving a unit positive charge along an infinitesimal distance  $\delta l$ ,

$$|E_l| \delta l = V_A - V_B = V - (V + \delta V) = -\delta V$$

$$\text{or } E = -\frac{\delta V}{\delta l}$$

- i. The electric field is in the direction in which the change in electrostatic potential decreases most. (This conclusion comes due to the negative sign of the above expression)

- ii. Magnitude of electric field is given by the change in the magnitude of electrostatic potential per unit displacement normal to the equipotential surface at the point.

19. The slope of Q-V graph of the capacitor gives capacitance, therefore, capacitor L has greater capacitance than K, thus the capacitance will be more for L. Since two capacitors are given the same voltage V, therefore energy stored in a capacitor will be given by the formula.

$$U = \frac{1}{2} CV^2 \text{ and for the same } V$$

$$U \propto C$$

$$\therefore C_L > C_K$$

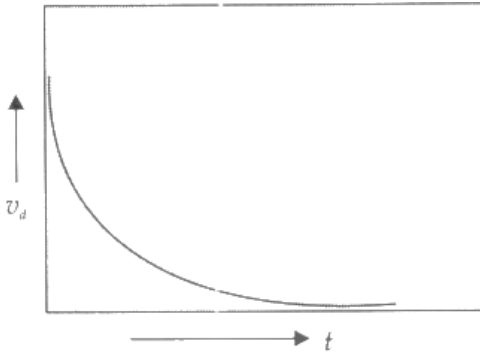
$$\Rightarrow U_L > U_k$$

$\Rightarrow$  Capacitor L will store more electrostatic energy.

20. Relaxation time is the time interval between two successive collisions of electrons in a conductor when current flows. It is the time taken for the drift velocity to decay  $\frac{1}{e}$  of its initial value.

As drift velocity increases, relaxation time decreases since the electrons move the distance in which they frequently collide faster.

Relaxation time = mean free path of electron / drift velocity of the electron.



When a potential difference  $V$  is applied across a conductor of length  $l$ , then drift speed of electron will result as :

$$v_d = \frac{eE\tau}{m}$$

$$= \frac{eV\tau}{lm} \because \left[ E = \frac{V}{l} \right]$$

The electric current through the conductor and drift speed are linked as  $I = neAv_d$  where,

$n$  = number density of electrons

$e$  = electronic charge

$A$  = area of cross-section

$v_d$  = electron drift speed

$$\therefore I = neA \left( \frac{eV\tau}{lm} \right)$$

$$\text{So, } \frac{V}{I} = \frac{ml}{ne^2\tau A}$$

At constant temperature :

$$\frac{V}{I} = R$$

$$\text{Hence, } R = \left( \frac{m}{ne^2\tau} \right) \times \frac{l}{A}$$

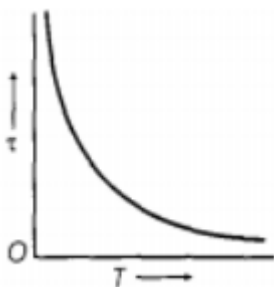
Comparing the above expression with

$$R = \rho \frac{l}{A}$$

where,  $\rho$  = specific resistance

$$\rho = \frac{m}{ne^2\tau}$$

21. Variation of resistivity ( $\rho$ ) with temperature ( $T$ ) is shown below:



**Explanation:** In semiconductor the number density of free electrons ( $\rho$ ) increases with increase in temperature ( $T$ ) and consequently the relaxation period decreases. But the effect of increase in  $\rho$  has higher impact than decrease of  $T$ . So, resistivity decreases with increase in temperature.

22. i. If a dipole is placed in an electric field, then in order to rotate it we have to do the work against electric field lines which can be found as:

$$\text{Work done in rotating the dipole, } W = \int_{\theta_1}^{\theta_2} \tau d\theta$$

If the dipole is turned from direction parallel to electric field to direction opposite to electric field, then angle  $\theta$  will change from 0 to  $\pi$ .

ii. We know that,  $\tau = pE \sin \theta$

If  $\theta = \pi/2$ , then  $\tau$  is maximum

i.e.  $\tau = pE \sin \frac{\pi}{2} \Rightarrow \tau = pE$  (maximum)

Maximum torque will be experienced by the dipole when its dipole moment is perpendicular to electric field lines.

23. Number of atoms in each piece of copper =  $\frac{6 \times 10^{23} \times 10}{63.5} = 9.45 \times 10^{22}$

Number of electrons transferred,  $n = \frac{1}{1000} \times 9.45 \times 10^{22}$

$n = 9.45 \times 10^{19}$

$\therefore$  Charges on each piece after transfer

$q_1 = q_2 = \pm ne = \pm 9.45 \times 10^{19} \times 1.6 \times 10^{-19}$

$= \pm 15.12C$

Given  $r = 0.1$  m

Thus,  $F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = 9 \times 10^9 \times \frac{(15.12)^2}{(0.1)^2}$

$= 2.06 \times 10^{14} N$

24. The work done in charging a capacitor is stored as its electrical potential energy.

Now considering a capacitor of capacitance  $C$ , initially whose two plates are uncharged, let  $Q$  and  $-Q$  are charges on the two plates and produces a uniform electric field,

$E = \frac{\sigma}{\epsilon_0}$  at any point between the plates ( $\sigma$  being surface charge density) and a potential difference

$V = \frac{q}{C} \dots(i)$

If an infinitesimal charge  $dq$  is transported in steps from negative charged plate to positive charged plate, till charges rises to  $+Q$  and  $-Q$ , then

work done,  $dW = dq \times V \dots(ii)$

From Eqs. (i) and (ii), we get

$dW = dq \left( \frac{q}{C} \right) \Rightarrow W = \int dW$

To charge the capacitor from 0 to  $Q$ , work done is stored as electrical potential energy  $U$  in the capacitor

$\therefore U = W = \int_0^Q \frac{q}{C} \times dq = \frac{Q^2}{2C}$

$\therefore U = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$  [ $\because Q = CV$ ]

OR

In series combination, charge on each capacitor always remains same.

i. All capacitors of capacitances  $6 \mu F$  are in series combination, then equivalent capacitance of this series combination is given by

$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$  or  $C' = \frac{C}{n} = \frac{6\mu F}{3} = 2\mu F$

Now this  $C'$  and another  $2\mu F$  capacitors are in parallel combination.

Hence, the equivalent capacitance of the whole circuit,

$C_{eq} = C' + 2\mu F = 2\mu F + 2\mu F$

$C_{eq} = 4 \mu F$

ii. Since,  $C'$  and  $2\mu F$  are in parallel combination, therefore both of them will get the same potential difference i.e. 6 V.

Now charge on  $C'$

$q' = C'V = (2\mu F) \times 6V = 12 \mu C$

The charge across each capacitor of capacitance  $6 \mu F$  is same and equal to charge across the series combination of them i.e.  $12\mu C$

Charge on  $2\mu F$  capacitor,  $q = CV = (2\mu F)(6 V) = 12\mu C$

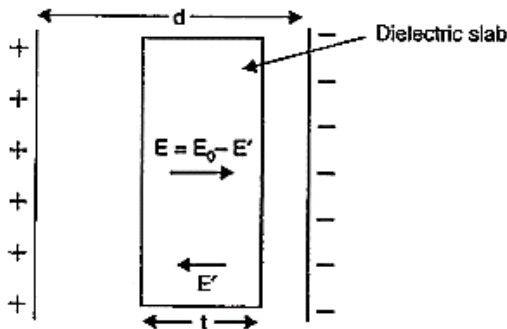
25. The potentiometer works on the principle that potential difference across any two points of a uniform current carrying conductor is directly proportional to the length between those two points.

- The area of cross-section has to be uniform to get a 'uniform wire' as per the principle of the potentiometer.
- The emf of the driving cell has to be greater than the emf of the primary cells as otherwise, no balance point would be obtained.

### Section D

26. Let A is the area of the two plates of the parallel plate capacitor and d is the separation between them. A dielectric slab of thickness  $t < d$  and area A is kept between the two plates. The total electric field inside the dielectric slab will be:

$E = \frac{E_0}{K} = E_0 - E'$  where  $E'$  is the opposite field developed inside the slab due to polarization of slab. Total potential difference between the plates,



$$V = E_0(d - t) + Et$$

$$= \frac{\sigma}{\epsilon_0}(d - t) + \frac{\sigma}{k\epsilon_0}t$$

$$= \frac{\sigma}{\epsilon_0} \left[ (d - t) + \frac{t}{k} \right]$$

$$V = \frac{q}{A\epsilon_0} \left[ (d - t) + \frac{t}{k} \right]$$

where q is the charge on each plate.

Since,  $C = \frac{q}{V}$

or  $C = \frac{q}{\frac{q}{A\epsilon_0} \left[ (d - t) + \frac{t}{k} \right]}$

or  $C = \frac{A\epsilon_0}{\left[ (d - t) + \frac{t}{k} \right]}$

27. i. As given in the question, energy of the  $6 \mu\text{F}$  capacitor is E. Let V be the potential difference along the capacitor of capacitance  $6 \mu\text{F}$ . From the mathematical formula,

Since,  $\frac{1}{2}CV^2 = E$

$$\frac{1}{2} \times 6 \times 10^{-6} \times V^2 = E \Rightarrow V^2 = \frac{E}{3} \times 10^6 \dots (i)$$

Since, potential is same for parallel connection of capacitors of capacitances of  $6 \mu\text{F}$  and  $12 \mu\text{F}$ . Hence the potential through  $12 \mu\text{F}$  capacitor is also V. Hence, energy of  $12 \mu\text{F}$  capacitor is

$$E_{12} = \frac{1}{2} \times 12 \times 10^{-6} \times V^2 \text{ [From Eq.(i)]}$$

$$= \frac{1}{2} \times 12 \times 10^{-6} \times \frac{E}{3} \times 10^6 = 2E$$

- ii. Since, charge remains constant in series, the charge on  $6 \mu\text{F}$  and  $12 \mu\text{F}$  capacitors combined will be equal to the net charge of the circuit and also the charge on  $3 \mu\text{F}$  capacitor.

Using the formula,  $Q = CV$ , we can write

$$\Rightarrow (6 + 12) \times 10^{-6} \times V = 3 \times 10^{-6} \times V'$$

[as equivalent capacitance of the parallel combination of  $6 \mu\text{F}$  and  $12 \mu\text{F} = (6 + 12) \mu\text{F}$ ]

$$\therefore V' = 6V$$

Squaring the above equation on both sides, we get

$$V'^2 = 36V^2 \Rightarrow V'^2 = 12E \times 10^6 \text{ [putting the value of } V^2 \text{ from eq(i)]}$$

$$\therefore E_3 = \frac{1}{2} \times 3 \times 10^{-6} \times 12E \times 10^6 = 18E \text{ [energy stored in } 3 \mu\text{F capacitor]}$$

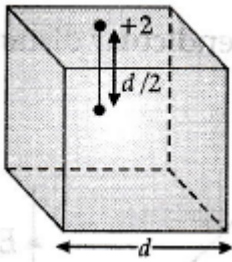
- iii. Total energy drawn from battery is  $E' = E + E_{12} + E_3 = E + 2E + 18E = 21E$

28. Electric flux: The electric flux may be defined as the number of electric lines of force crossing through a surface normal to the surface. It can be measured as the surface integral of the electric field over that surface, i.e.

$$\phi = \int_s \vec{E} \cdot d\vec{s}$$

Electric flux  $\phi$  is a scalar quantity.

Now to calculate the electric flux passing through the square of side  $d$ , draw a cube of side  $d$  such that it completely encloses the charge  $q$ . Now by using Gauss's law.



Total flux through the all the six surfaces of a cube is given as

$$\phi_{\text{total}} = 6 \times \phi_{\text{square face}} = \frac{\text{total charge enclosed}}{\epsilon_0}$$

$$\Rightarrow 6\phi_{\text{square}} = \frac{q}{\epsilon_0}$$

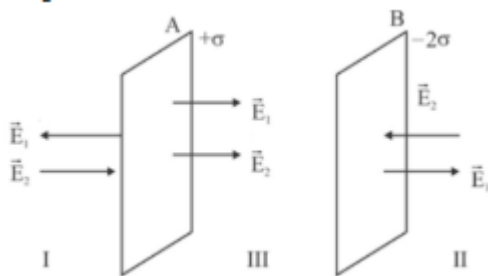
$$\Rightarrow \phi_{\text{square face}} = \frac{q}{6\epsilon_0}$$

Hence the flux through the square of side  $d$  with charge  $q$  at a distance  $d/2$  directly above the head is  $q/6\epsilon_0$ .

If a charge is now moved to the distance  $d$  from the center of square and side of the square is doubled, then electric flux remains unchanged because electric flux in a closed surface depends only on the amount of charge contained inside the closed surface and is independent of the distance of charge.

OR

Consider two infinite plane sheets A and B separated by a distance  $d$  having surface charge densities  $+\sigma$  and  $-2\sigma$  respectively as shown in the figure. Let  $\vec{E}_1$  and  $\vec{E}_2$  be the electric field intensities due to sheets A and B. Magnitude of Electric Fields  $E_1 = \frac{\sigma}{2\epsilon_0}$  and  $E_2 = \frac{\sigma}{\epsilon_0}$



i. Electric field in the region left of the first sheet,

$$E_I = -\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{\epsilon_0}$$

$$E_I = +\frac{\sigma}{2\epsilon_0}$$

It's direction is towards right.

ii. Electric field in the region to the right of second sheet,

$$E_{II} = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{\epsilon_0}$$

$$E_{II} = -\frac{\sigma}{2\epsilon_0}$$

It's direction is towards left.

iii. Electric field between the two sheets,

$$E_{III} = \frac{\sigma}{\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

$$E_{III} = \frac{3\sigma}{2\epsilon_0}$$

Here, electric field is towards the right.

29.

Conductor A (solid wire $R_A$ )	Conductor B (hollow tube $R_B$ )
$l_1 = l$	$l_2 = l$ (Given)
$A_1 = \pi r_1^2$	$A_2 = \pi r_2^2 - \pi r_1^2$
$r_1 = \frac{1}{2} \text{mm} = 0.5 \times 10^{-3} \text{m}$	$r_2 = \frac{2}{2} \text{mm} = 1 \times 10^{-3} \text{m}$
$\rho_1 = \rho$	$\rho_2 = \rho$

$$\Rightarrow \frac{R_A}{R_B} = \frac{\frac{\rho_1 l_1}{A_1}}{\frac{\rho_2 l_2}{A_2}}$$

$$\Rightarrow \frac{R_A}{R_B} = \frac{\rho_1 l_1}{A_1} \times \frac{A_2}{\rho_2 l_2}$$

$$\Rightarrow \frac{R_A}{R_B} = \frac{\rho l}{A_1} \times \frac{A_2}{\rho l}$$

$$\therefore \frac{R_A}{R_B} = \frac{A_2}{A_1} = \frac{\pi r_2^2 - \pi r_1^2}{\pi r_1^2} = \frac{\pi(r_2^2 - r_1^2)}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right)^2 - 1$$

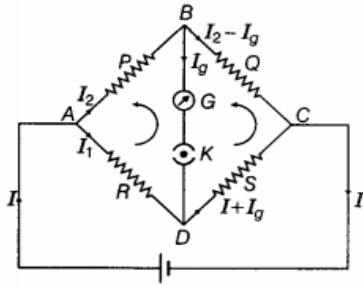
$$(1 \times 10^{-3} / 0.5 \times 10^{-3})^2 - 1$$

$$\therefore R_A : R_B = 3 : 1$$

OR

Applying Kirchhoff's loop law to close loop ABDA, we get

$$I_1 R - I_g G - I_2 P = 0 \dots (i)$$



Here, G is the resistance of the galvanometer.

Applying Kirchhoff's loop law in the closed loop BDCB, we get

$$I_g G + (I_1 + I_g) S - (I_2 - I_g) Q = 0 \dots (ii)$$

When the Wheatstone bridge is balanced, no current flows through the galvanometer,

i.e.  $I_g = 0$

$\therefore$  From Eq. (i), we get

$$I_1 R - I_2 P = 0 \Rightarrow I_1 R = I_2 P$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{P}{R} \dots (iii)$$

Similarly, from Eq. (ii), we get

$$I_1 S - I_2 Q = 0$$

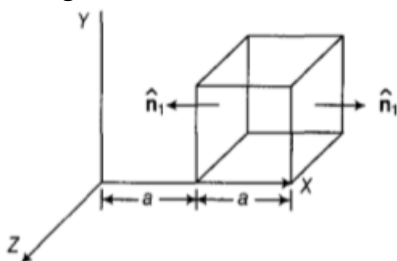
$$\Rightarrow I_1 S = I_2 Q = 0 \dots (iv)$$

From Eqs. (iii) and (iv), we get

$$\frac{P}{R} = \frac{Q}{S} \Rightarrow \frac{P}{Q} = \frac{R}{S}$$

This is the required balance condition in a Wheatstone bridge arrangement and also helps us to determine unknown resistance.

30. Gauss's Law: It states that the total electric flux crossing through a closed surface is equal to times the total charge contained inside the closed surface.



$$E = Cx \hat{i}$$

$$\text{Electric field at left face of cube} = E_1 = Ca \hat{i}$$

$$\text{Electric field at the right surface} = E_2 = 2Ca \hat{i}$$

$$\text{Electric flux through left surface of cube} = E_1 A \cos 180 = -E_1 A = -Ca \cdot a^2 = -Ca^3 \text{ units}$$

$$\text{Electric flux through right surface of cube} = E_2 A \cos 0 = E_2 A = 2Ca \cdot a^2 = 2Ca^3 \text{ units}$$



i. Net flux through the cube =  $2Ca^3 - Ca^3 = Ca^3$  units

ii. Using Gauss law net flux inside the cube =  $\frac{q}{\epsilon_0}$

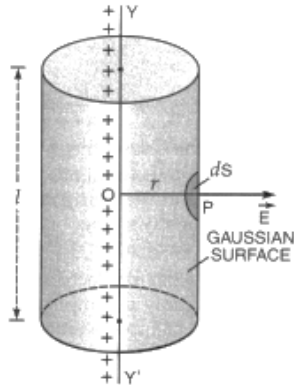
From (i),

$$q = Ca^3 \epsilon_0$$

This is the net charge inside the cube.

### Section E

31. a. Consider a thin infinitely long straight line charge having a uniform linear charge density  $\lambda$  placed along YY'. Draw a cylindrical surface of radius  $r$  and length  $l$  about the line charge as its axis.



If  $E$  is the magnitude of electric field at point P, then electric flux through the gaussian surface is given by

$$\phi = E \times \text{area of the curved surface of a cylinder of cylinder radius } r \text{ and length } l$$

or

$$\phi = E \times 2\pi r l$$

According to Gauss' theorem, we have  $\phi = \frac{q}{\epsilon_0}$

Now, charge enclosed by the gaussian surface,  $q = \lambda l$

$$\therefore \phi = \frac{\lambda l}{\epsilon_0}$$

Thus,

$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$\text{or } E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$

- b. Electric field at a distance  $r$  from the line charge,

$$E = \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{r}$$

To calculate force on charge  $-q$  at point A:

$$\text{Here, } OA = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$$

Electric field at point A,

$$E_1 = \frac{1}{2\pi \times 8.854 \times 10^{-12}} \times \frac{4.5 \times 10^{-4}}{2.5 \times 10^{-2}}$$

$$= 3.24 \times 10^8 \text{ NC}^{-1}$$

Force on charge  $-q$  at point A,  $F_1 = qE_1 = 5 \times 10^{-9} \times 3.24 \times 10^8 = 1.62 \text{ N}$  (towards the line charge)

To calculate force on charge  $+q$  at point B:

$$\text{Here, } OB = 2.5 \times 10^{-2} + 2 \times 10^{-3} = 2.7 \times 10^{-2} \text{ m}$$

Electric field at point B,

$$E_2 = \frac{1}{2\pi \times 8.854 \times 10^{-12}} \times \frac{4.5 \times 10^{-4}}{2.7 \times 10^{-2}}$$

$$= 3 \times 10^8 \text{ NC}^{-1}$$

Force on charge  $+q$  at point B,

$$F_2 = qE_2 = 5 \times 10^{-9} \times 3 \times 10^8 = 1.5 \text{ N}$$
 (away from the line charge)

Hence, net force on electric dipole,

$$F = F_1 - F_2 = 1.62 - 1.5 = 0.12 \text{ N}$$
 (towards the line charge)

OR

- a. Since the work done depends on the final arrangement of the charges, and not on how they are put together, we calculate work needed for one way of putting the charges at A, B, C and D. Suppose, first the

charge  $+q$  is brought to A, and then the charges  $-q$ ,  $+q$ , and  $-q$  are brought to B, C, and D, respectively. The total work needed can be calculated in steps:

i. Work needed to bring charge  $+q$  to A when no charge is present elsewhere: this is zero.

ii. Work needed to bring  $-q$  to B when  $+q$  is at A. This is given by (charge at B)  $\times$  (electrostatic potential at B due to charge  $+q$  at A)

$$= -q \times \left( \frac{q}{4\pi\epsilon_0 d} \right) = -\frac{q^2}{4\pi\epsilon_0 d}$$

iii. Work needed to bring charge  $+q$  to C when  $+q$  is at A and  $-q$  is at B. This is given by (charge at C)  $\times$  (potential at C due to charges at A and B)

$$= +q \left( \frac{+q}{4\pi\epsilon_0 d\sqrt{2}} + \frac{-q}{4\pi\epsilon_0 d} \right)$$

$$= \frac{-q^2}{4\pi\epsilon_0 d} \left( 1 - \frac{1}{\sqrt{2}} \right)$$

iv. Work needed to bring  $-q$  to D when  $+q$  at A,  $-q$  at B, and  $+q$  at C. This is given by (charge at D)  $\times$  (potential at D due to charges at A, B, and C)

$$= -q \left( \frac{+q}{4\pi\epsilon_0 d} + \frac{-q}{4\pi\epsilon_0 d\sqrt{2}} + \frac{q}{4\pi\epsilon_0 d} \right)$$

$$= \frac{-q^2}{4\pi\epsilon_0 d} \left( 2 - \frac{1}{\sqrt{2}} \right)$$

Add the work done in steps (i), (ii), (iii), and (iv). The total work required is

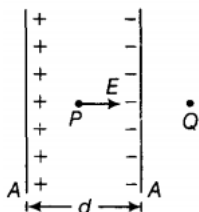
$$= \frac{-q^2}{4\pi\epsilon_0 d} \left\{ (0) + (1) + \left( 1 - \frac{1}{\sqrt{2}} \right) + \left( 2 - \frac{1}{\sqrt{2}} \right) \right\}$$

$$= \frac{-q^2}{4\pi\epsilon_0 d} (4 - \sqrt{2})$$

The work done depends only on the arrangement of the charges, and not how they are assembled. By definition, this is the total electrostatic energy of the charges.

b. The extra work necessary to bring a charge  $q_0$  to point E when the four charges are at A, B, C, and D is  $q_0 \times$  (electrostatic potential at E due to the charges at A, B, C, and D). The electrostatic potential at E is clearly zero since potential due to A and C is cancelled by that due to B and D. Hence, no work is required to bring any charge to point E. Also, it can be said that the work done over a closed surface is zero. (charges are opposite in corners so work done during one cycle cancel out by another cycle) hence work done is zero.

32. a. Consider the figure shown below:



i. Electric field due to the plate of the positive charge of charge density  $+\sigma$  at point P, is given by  $E_1 = \sigma/2\epsilon_0$

Magnitude of electric field due to the other plate of negative charge density  $-\sigma$ , is given by

$$E_2 = -\sigma/2\epsilon_0$$

In , the inner region between the plates 1 and 2 , electric field due to the two charged plates add up, is given by

$$E_{\text{net}} = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

Outside the plate, electric field will be equal to zero because of the opposite directions of the electric fields  $E_1$  and  $E_2$  there.

ii. Potential difference between the plates of the capacitor is given by

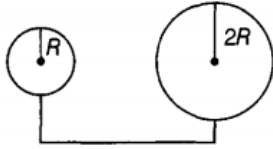
$$V = Ed = \sigma d/\epsilon_0 \quad (\because E = \sigma/\epsilon_0)$$

iii. Capacitance of the capacitor is given by

( $\because Q = CV$ )

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d/\epsilon_0} = \frac{\epsilon_0 A}{d}$$

b. Consider the figure shown below:



Potential at the surface of the sphere of radius R,

$$= \frac{kq}{R} \quad [\because q = \sigma \times 4\pi R^2]$$

$$= \frac{k\sigma 4\pi R^2}{R} = \sigma k 4\pi R = 4k\sigma\pi R$$

Potential at the surface of the second sphere of radius twice the previous one i.e. 2R,

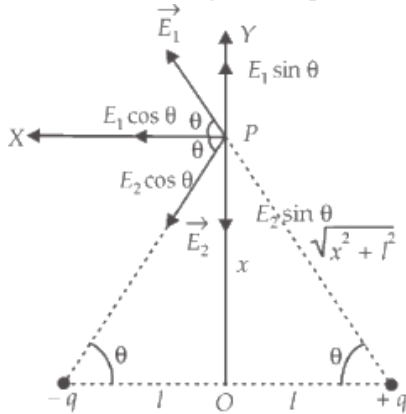
$$= \frac{kq}{2R} \quad [\because q = \sigma \times 4\pi(2R)^2 = 16\sigma\pi R^2]$$

$$= \frac{k16\sigma\pi R^2}{2R} = 8k\sigma\pi R$$

We know that charge always flows from the higher potential surface to lower potential surface. Since the potential of the bigger sphere is more, so charge will flow from sphere of radius 2R to the sphere of radius R after connecting both the spheres by a conducting wire

OR

i. Consider a point P lying on the perpendicular bisector of the line joining the two charges at a distance x from the midpoint O (fig). Let  $\vec{E}_1$  and  $\vec{E}_2$  be the field intensities at P due to +q and -q charges.  $\vec{E}_1$  is directed away from the positive charge (+q) and  $\vec{E}_2$  is directed towards the negative charge (-q). The distance of charge from point P is  $\sqrt{x^2 + l^2}$  (by Pythagoras theorem).



Now, the magnitude of each field is given by

$$E_1 = E_2 = \frac{q}{4\pi\epsilon_0(x^2+l^2)}$$

On resolving each field along X and Y axes and adding their respective components, we find that their Y-components cancel out and X-components add up. The resultant vector  $\vec{E}$  is in the direction opposite to the dipole moment vector.

$$E = E_1 \cos\theta + E_2 \cos\theta = 2E_1 \cos\theta$$

$$= \frac{2q}{4\pi\epsilon_0(x^2+l^2)} \cdot \frac{l}{(x^2+l^2)^{1/2}} \quad \left[ \because \cos\theta = \frac{l}{(x^2+l^2)^{1/2}} \right]$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{(x^2+l^2)^{3/2}}$$

In vector form,

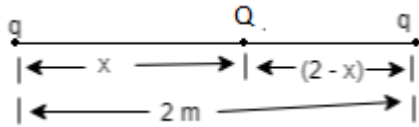
$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{(-\vec{p})}{(x^2+l^2)^{3/2}}$$

The negative sign indicates  $\vec{E}$  is opposite in direction to  $\vec{p}$ .

If  $x \gg l$ , then we have

$$\vec{E} = -\frac{\vec{p}}{4\pi\epsilon_0 x^3}$$

ii. The system is in equilibrium, therefore, the net force on each charge of the system will be zero.



For the total force on 'Q' to be zero,

$$\frac{1}{4\pi\epsilon_0} \frac{qQ}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{(2-x)^2}$$

$$\Rightarrow x = 2 - x$$

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = 1\text{m}$$

For the equilibrium of charge q, the nature of charge Q must be opposite to the nature of charge q.

33. "Drift velocity": Drift velocity is define as the average velocity with which the electrons get drifted towards positive end of the conductor under the influence of an external electric field. it is denoted by Vd.

Thus,  $V_d = l/t = (U+V)/2$

The Free electron density of a conductor be "n" electrons/metre <sup>3</sup>.

i.e 1m<sup>3</sup> volume containing = n free electron

Hence,

(AL) m<sup>3</sup> Volume containing (N) = n.(AL) electrons

Thus, amount of charge flows through any cross section of a conductor, Q = N.e

$$I = n.e.A(l/t)$$

{By the definition

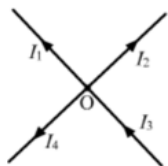
Drift velocity, Vd = l/t }

$$I = n.eA(V_d)$$

OR

i. **Kirchhoff' First law** states that "The sum of the currents flowing towards a junction is equal to the sum of current leaving the junction."

This is in accordance with the conservation of charge which is the basis of Kirchhoff's current rule.

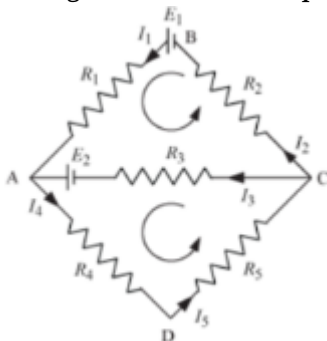


Here I<sub>1</sub>, I<sub>2</sub>, I<sub>3</sub>, and I<sub>4</sub> are the currents flowing through the respective wires.

Convention: The current flowing towards the junction is taken as positive and the current flowing away from the junction is taken as negative.

$$I_3 + (-I_1) + (-I_2) + (-I_4) = 0$$

**Kirchhoff's Second Law or Loop Rule** states that in a closed loop, the algebraic sum of the emf is equal to the algebraic sum of the products of the resistances and currents flowing through them.



For the closed loop BACB:

$$E_1 - E_2 = I_1R_1 + I_2R_2 - I_3R_3$$

For the closed loop CADC:

This law is based on the law of conservation of energy.

ii. Total resistance of network = r/3

internal resistance = r

Total circuit resistance  $R = r + r/3 = 4r/3$

a. Current drawn from cell,  $I = \frac{E}{4r/3} = \frac{3E}{4r}$

b. Power consumed in the network  $= I^2 R = \frac{9E^2}{16r^2} \times \frac{4r}{3} = \frac{3E^2}{4r}$  units.